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Identifying Inefficiencies in Multiple Output-Multiple Input Organizations

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SSM Working Paper: #1316-82
February, 1982

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Functional Areas:

- Organization performance
- Accounting (Auditing)
- Nonprofit and public Sector Organizations

Methodological Areas:

- Mathematical Programming

Abstract

Data Envelopment Analysis (DEA), a new methodology based on linear programming concepts, provides an approach to evaluate relative technical efficiency of nonprofit organizations 1) which have multiple outputs and inputs and 2) where the efficient production function is not specifiable with precision. This paper evaluates the reliability of DEA, compared with use of ratio analysis and basic regression analysis for efficiency measurement.

DEA, ratio analysis, and regression analysis are applied to an artificial data, in which the efficient and inefficient units are known. Without knowledge of the technology, DEA accurately identifies certain inefficient Decision Making units (DMU's) when outputs and input are properly specified. In contrast, the ratio analysis and regression analysis techniques are found to be less reliable for identifying inefficient DMU's. The strengths and limitations of DEA are further elaborated to anticipate issues that may arise in subsequent field applications.

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I. Introduction

Date Envelopment Analysis (DEA) is a new efficiency measurement methodology developed by A. Charnes, W. W. Cooper, and E. Rhodes [9] [10] and [11]. DEA is designed to measure relative technical efficiency of Decision Making Units (DMU) which use multiples inputs to produce multiple outputs where the underlying production function is not known with any precision. DEA has already been applied to several types of organizations including education [5] [6], health care [4] [22], Navy recruiting centers [17], and criminal court systems [16]. Some of these applications have been helpful to managers of these organizations, i.e., DEA results have been used to implement changes to improve efficiency. Nevertheless, the validity and reliability of DEA in its locating of inefficient DMU's has not been evaluated in these studies, in part because the identity of the truly inefficient units were not known. This paper attempts to evaluate DEA and other efficiency measurement techniques through a simulated application to an artificial data base where the efficient and inefficient DMU's are known with certainty. The objective is to compare the accuracy and evaluate the validity of these techniques in locating inefficient DMU's.[1] This process leads to some strong conclusions about the relative strengths and weaknesses of these techniques and provides certain insights about DEA that may be useful for future applications to real data sets.

In this paper, the efficiency measurement techniques that will be evaluated are DEA, basic regression analysis, and ratio analysis. Regression analysis and ratio analysis were selected because they represent approaches which are relatively widely known and accessible to managers.[2] More sophisticated regression techniques such as the flexible functional forms like the translog function are not considered here but also require similar

evaluation and validation for use in efficiency measurement as discussed in Sherman [22], and Banker, Conrad, & Strauss [4]. In other words, this paper is aimed at comparing DEA with techniques that are already widely used rather than new methodologies which may also be useful for efficiency measurement.

The following section describes the data base constructed to test alternative efficiency measurement methodologies. Section 3 describes the version of DEA that will be used to evaluate the efficiency of these DMU's. In Section 4, the result of applying DEA, simple regression analysis, and ratio analysis are compared and summarized. Section 5 considers other interpretation that are available from DEA as well as certain of its limitations that are apparent from this simulation. The final section contains a brief discussion of other areas of research required to further validate and develop the capabilities of DEA and other efficiency measurement techniques.

2. Artificial Data Set - development and specifications

The artificial data set is constructed by defining a hypothetical "known" technology which applies to all Decision Making Units (DMU's) and defines efficient input-output relationships within these DMUs' industry. Inefficiencies are introduced for certain DMU's and take the form of excess inputs used for the output level attained. Hence, a DMU that achieves its output level by using the amount of inputs required based on the specified technology is efficient and a DMU that uses more inputs than are required by this technology is inefficient. To make the inputs and outputs easier to recognize, they are referred to and labelled in the context of a hospital, which is one type of organization which uses multiple inputs to produce multiple outputs and where techniques like DEA may prove beneficial. [3]

The set of artificial hospital data generated for our simulation consisted of three outputs produced with three inputs during a one year period of time as follows:

Outputs		Inputs	
y_1 :	Regular patient* care/year (patients treated in one year with average level of inputs for treatment)	x_1:	Staff utilized in terms of full-time equivalents, i.e., (FTE's)/year
y_2 :	Severe patient* care/year (patients treated in one year with severe illness requiring higher input levels than regular patients for more complex treatment).	x_2:	Number of hospital bed days available/year
y_3 :	Teaching of residents and interns/year (number of individuals receiving one year of training)	x_3:	Supplies in terms of dollar cost/year

*measured in terms of number of patients treated

A linear input-output model was used to specify the known technology, and it was assumed to be applicable to all hospitals. That is, deviations from this structure represent inefficiencies which the DEA analysis--or any other analysis that might be used--should be able to detect.

For convenience of reference all details of the model and resulting data utilized are collected together in the Appendix. The model and the input output relationships (and data used) to develop the model parameters are given in Exhibit 1. These production relationships are assumed to hold for all volume levels of operations for all hospitals each of which, however, may use them efficiently or inefficiently. Input costs per unit are also fixed at the same amounts for all hospitals and remain the same at all levels of operation so that the resulting production activity can be converted in to common dollar units. It is also assumed that all hospitals are subject to the same "production function" which has constant returns to scale in all

outputs. This provides the underlying structure which we shall henceforth refer to as the "structural model."

Via this "structural model" as represented in exhibit 1, data were developed for an assumed set of 15 hospitals based on arbitrary mixes of outputs. The related inputs required were derived from the model and the outputs. The resulting data base which we shall henceforth use is shown in Exhibit 2. The first seven hospitals, H1-H7, are efficient; i.e., the inputs and outputs are those required in the structural model. The data generated for the next eight hospitals, H8-H15, were developed by altering the numerical values to portray various inefficiencies. The idea of course is to test the ability of DEA and other methodologies to identify such inefficiencies. These efficiency measurement techniques would be accurate--at least as far as classification is concerned--if they isolated H8 through H15 as inefficient in this applications.

The specific inefficiencies in H8 through H15 are designated by the circled ◯ values in Exhibit 2. That is, these circled values refer to the specifically inefficient elements in supposed managerial uses for these hospitals relative to what the production function requires for the outputs they have achieved. Exhibit 3 then presents an example of how the data for the efficient hospital, H1, and the inefficient hospital, H15, were calculated. H15 is represented as a) inefficient in its use of inputs to produce regular patient care and b) efficient in its use of inputs to produce severe patient care outputs and to provide training (teaching) outputs.

To effect comparisons such as we shall be undertaking in a multiple output context, it is easiest to proceed from the other side and to locate all inefficiences in the inputs used to produce whatever output levels that were attained. We can do this because of the linear relationship we are assuming since, as Fare and Lovel [12] have shown, the kind of efficiency measures we

are applying will give the same result from the output or the input side when the relations are linear.

The model underlying Exhibit 2 can be formalized as follows:

$$x_{ij} = \sum_{r=1}^{3} a_{irj}\, y_{rj} \qquad (1)$$

where x_{ij} = amount of input i used per year by hospital j

y_{rj} = amount of output r produced per year by hospital j

a_{irj} = amount of input i used per unit of output r by hospital j during the year.

We are positing an efficient set of a_{irj}'s which are the same for every hospital. However, we retain the index j because in some cases we will assign values $\hat{a}_{irj} > a_{irj}$ for some i and r to represent managerial (= hospital) inefficiencies which yield values

$$\hat{x}_{ij} = \sum_{r=1}^{3} \hat{a}_{irj}\, y_{rj} \qquad (2)$$

with $\hat{x}_{ij} > x_{ij}$ when inefficiencies are present.

The efficient a_{ir} values are given, free of any of the j = 1,..., 15 hospital identification subscripts, in Exhibit 1. These values are the same for all hospitals so that a_{11} = .004 FTE/patient represents the efficient labor requirement in full time equivalent units per regular patient. Similarly a_{12} = .005 FTE/patient represents the efficient requirement for a severe patient and a_{13} = .03 FTE/training unit represents the efficient requirement to train one new resident/intern during a year.

Analogous remarks apply to the values a_{21} = 7 bed days/patient, and a_{22} = 9 bed days/patient for regular and severe patients, respectively, shown in the Bed Days column of Exhibit 1. The blank shown in the row for

Training Units in this column means that $a_{23} = 0$ applies. That is, no bed days enter into the training outputs.

Finally, $a_{31} = \$20$/patient and $a_{32} = \$30$/patient represent the efficient level of supplies required per regular and severe patients, respectively, while $a_{33} = \$500$/training unit is the coefficient for efficient training operations in output $r = 3$. Putting this $i = 3$ input in dollar units avoids the detail that would otherwise be needed to identify the different types of supplies that would be required for teaching and for different types of patient treatments.

The final column of Exhibit 1 represents the efficient costs obtained from

$$c_r = \sum_{i=1}^{3} k_i \, a_{ir} \tag{3}$$

where we have omitted the index j for hospital identification because only efficient costs are being considered. Here k_i represents the cost of the i^{th} input requirement for the r^{th} output under efficient operations.

Via the data shown in Exhibit 1 (reflected immediately below the table in Exhibit 1)

$$\begin{aligned} k_1 &= \$10{,}000/\text{FTE} \\ k_2 &= \$10/\text{bed day} \\ k_3 &= \$1/\text{supply unit (already reflected in \$ in the data base – Exhibit 3.1)} \end{aligned} \tag{4}$$

from which we obtain

$$\begin{aligned} c_1 &= k_1a_{11} + k_2a_{21} + k_3a_{31} = \$130/\text{regular patient} \\ c_2 &= k_1a_{12} + k_2a_{22} + k_3a_{32} = \$170/\text{severe patient} \\ c_3 &= k_1a_{13} + k_2a_{23} + 1_3a_{33} = \$500 \text{ training unit.} \end{aligned} \tag{5}$$

These are the formulas used at the bottom of Exhibit 1 to produce the efficient cost of outputs shown in the last column at the top.

We now turn to Exhibit 2 which reflects the composition of inefficient and efficient hospitals included in our data base. Actual inputs per unit output

used by each hospital are listed in Exhibit 2, columns 9-16 with inefficient input levels per unit of output noted by the ○. Column 17 reflects the actual vacancy rate (% of unused bed days available during the year). An efficient hospital is expected to have a 5% vacancy rate.

We develop the actual inputs used for each hospital by selecting an arbitrary set of outputs: Teaching units per year is reflected in col. 6, regular paitents treated during the year are in col. 7, and severe patients treated during the year are in col. 8. Other ways of summarizing patient care outputs are included in columns 4 and 5. Column 4 reflects total patients which is the sum of col. 7 and col. 8. Column 5 reflects the percentage (%) of severe patients treated which is based on (col. 8) ÷ (col. 4) x (100). We develop this percentage output measure because it reflects output data in the form which is most readily accessible in many real data sets and it will allow us to consider how such data may be used in our DEA efficiency evaluation of these DMU's. The inputs used by each hospital to produce the outputs in columns 6, 7, and 8 are reflected in columns 1, 2, and 3. Column 1 contains the full time equivalents (FTE's) of labor years used by the hospital during one year. Column 2 has the bed days available per year to treat patients. Column 3 gives the supply dollars used per year. We clarify how these input levels were calculated by example in Exhibit 3.

Exhibit 3 illustrates how H1, an efficient DMU, and H15, an inefficient DMU, data were constructed. H1 is efficient and therefore used the same inputs per unit outputs as the structural model in Exhibit 1. During the year, H1 provides care for 3000 regular patients, 2000 severe patients, and 50 training units of service. It, therefore, utilized (.004)(3000) + (.005)(2000) + (.03)(50) = 23.5 FTE's in that year. H15 produced the same outputs as H1 but was inefficient in its use of certain inputs. It used .005

FTE's/regular patient, while it adhered to the structural model FTE usage rates for severe patients (.005 FTE's/patient) and training (.03 FTE's/training unit). H15 therefore used (.005)(3000) + (.005)(2000) + (.03)(05) = 26.5 FTE's/year to produce the same outputs. Similarly, H15 is <u>inefficient</u> in the number of bed days used and supply dollars used per regular patient and is <u>efficient</u> in the amount of bed days and supply dollars consumered for severe patients and for supply dollars used for teaching outputs. Bed day and FTE's and supply dollar inputs are also calculated in Exhibit 3 to further illustrate the way the data base was constructed.

The number of FTE's, bed-days, and supply dollars inputs were calculated as illustrated in Exhibit 3 for each hospital based on the arbitrarily assigned output mix of regular patients, severe patients and training units and the actual efficient or inefficient input per unit output rate reflected in Exhibit 2.

Certain relationships posited in the structural model are generally not known, like the actual amount of staff time and supplies that are required to support each intern or residentat a hospital. We nevertheless explicitly introduce these relationships in the simulation to determine if the efficiency measurement techniques we will apply can uncover them using only the resulting input and output data. Before proceeding, it should be noted that when the underlying structural model is known, the determination of which DMU's are inefficient can be directly determined and techniques such as we will be considering would be unnecessary for purposes of efficiency evaluation.

3. <u>The DEA Model</u>

The Charnes, Cooper and Rhodes (CCR) [9] [10] data envelopment analysis (DEA) technique will be applied to the artificial data set using the following formulation which is a linear programming format of the fractional programming form of DEA described in CCR [9].[4]

Objective

$$\text{Max } h_o^* = \sum_{r=1}^{3} u_r y_{ro}$$

where o is the DMU being evaluated in the set of o = 1,...15 DMU's.

$$\text{Subject to} \quad 0 \geq \sum_{r=1}^{3} u_r y_{rj} - \sum_{i=1}^{3} v_i x_{ij}; \quad j = 1, \ldots, 15$$

$$1 = \sum_{i=1}^{3} v_i x_{io} \tag{6}$$

$$0 < u_r, v_i; \quad r = 1, 2, 3 \quad i = 1, 2, 3$$

Data: Outputs: y_{rj} = observed amount of r^{th} output for j^{th} DMU

Inputs: x_{ij} = observed amount of i^{th} input for j^{th} DMU

This application of linear programming is designed for an ex post evaluation of how efficient each DMU was in its use of inputs (x_i) to produce outputs (y_i) without explicit knowledge of the input output relationship it used. The weights in the form of u_r, v_i are also not known or given a priori. They are, instead, calculated as (u_r, v_i) values to be assigned to each input and output in order to maximize h_o* value for the DMU being evaluated (DMU_o).

Particular attention might be called to the positivity conditions on the variables, which CCR ensure by introducing the conditions

$$\varepsilon > u_r, \ \varepsilon < v_i \text{ for all r and i} \tag{7}$$

where $\varepsilon > 0$ is a small constant which is so small that it cannot otherwise disturb any solution involving only real numbers. We shall use the value $\varepsilon = .001$ in this discussion for numerical convenience. Although still smaller values may be used, a series of checks need to be made in any case to ensure that the numerical value assigned to ε does not alter the analysis and conclusions.[5]

With the introduction of the ε value, the efficiency rating (E) of DMU_o may be represented as

$$E_o = h_o^* - \varepsilon \left[\sum_{r=1}^{3} s_r^{-*} + \sum_{i=1}^{3} s_i^{+*} \right] \tag{8}$$

where s_r^{-*} and s_r^{+*} represent the negative and positive slack corresponding to outputs and inputs in the optimal linear programming solution related to DMU_o. Thus, applying DEA to a set of DMU's results in an efficiency ratio for each DMU of E = 1 indicating it is relatively efficient or E < 1 indicating it is relatively inefficient.[6]

The following analysis proceeds with the above interpretation of the efficiency rating (E). In the subsequent sections, the implication of this E value and the constraint that $\varepsilon > 0$ are reconsidered in the context of further interpretations of the DEA results.

Section 4 - Results of Alternative Efficiency Measurement Techniques: DEA, Ratio Analysis, Basic Regression Analysis

DEA, simple forms of regression analysis, and ratio analysis are applied to the artificial data set in Exhibit 2 to consider how each methodology performs in locating inefficient units.

Exhibit 2

Constructed Data Base

( ) = Inefficient use of inputs compared to a structural model of efficient input-output relationship described in Ex. 1

	Inputs					Outputs			Actual Inputs Used: FTE		Bed days		Supply $		Training		
	FTE	Bed Days	Supply $'s	Total* Pat.s	% Sev.** Pat.s	Tesch. Units	Reg. Pat.s	Sev. Pat.s	Reg. Pat.s	Sev. Pat.s	Reg. Pat.s	Sev. Pat.s	Reg. Pat.s	Sev. Pat.s	Supply $'s	FTE	Vacancy Rate
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
H1	23.5	41050	$130,000	5000	40	50	3000	2000	.004	.005	7	9	20	30	200	.03	.05
H2	24.5	43160	140,000	5000	60	50	2000	3000	.004	.005	7	9	20	30	200	.03	.05
H3	26.0	43160	150,000	5000	60	100	2000	3000	.004	.005	7	9	20	30	200	.03	.05
H4	25.0	41050	140,000	5000	40	100	3000	2000	.004	.005	7	9	20	30	200	.03	.05
H5	28.5	50530	160,000	6000	50	50	3000	3000	.004	.005	7	9	20	30	200	.03	.05
H6	36.0	62105	210,000	7000	71	100	2000	5000	.004	.005	7	9	20	30	200	.03	.05
H7	51.5	92630	270,000	12000	17	50	10000	2000	.004	.005	7	9	20	30	200	.03	.05
H8	25.0	49475	140,000	5000	14	100	3000	2000	.004	.005	7	9	20	30	200	.03	.05
H9	24.5	43160	165,000	5000	60	50	2000	3000	.004	.005	9	10	20	30	200	.03	.05
H10	77.0	92630	340,000	12000	17	100	10000	2000	.006	.007	7	9	25	35	200	.03	.05
H11	44.5	65260	265,000	8000	38	50	5000	3000	.005	.006	7	9	25	35	200	.03	.05
H12	30.0	60000	170,000	6000	50	100	3000	3000	.004	.005	7	9	30	35	200	.03	.05
H13	43.5	81110	245,000	9000	56	50	4000	5000	.004	.005	7	9	20	30	200	.03	.20
H14	30.0	60000	170,000	6000	50	100	3000	3000	.004	.005	9	9	20	30	300	.05	.10
H15	26.5	47370	160,000	5000	40	50	3000	4000	.005	.005	9	9	30	30	200	.03	.10

* Total Patients = Col. 7 + Col. 8

** % Severe = Col. 8 ÷ Col. 4

Note: Outputs (Col.s 4 - 8) and input usage (Col.s 9 - 17) are arbitrarily assigned so that H1 - H7 are efficient based on the production model in Ex. 1 and H8 - H15 are inefficient based on the same model. Inputs (Col. 1, 2, 3) are derived from Col.s 4 - 17 as indicated in Ex. 3

Exhibit 3

Appendix –

Example of Construction of Data Base for Hospitals H1 (efficient) and H15 (inefficient)

	H1 Efficient	H15 Inefficient	Difference Between H1 and H15	Inputs Required for Each Unit of the Related Output: FTE/yr. H1		FTE/yr. H15	Bed days/yr. H1		Bed days/yr. H15	Supply $'s/yr. H1		Supply $'s/yr. H15	Vacancy* Rate during the year. H1	Vacancy* Rate during the year. H15
Outputs (identical for H1 and H15)														
Y_1 Regular patients/yr.	3000	3000	–	.004	≠	.005	7	≠	9	$20	≠	$30		
Y_2 Severe patients/yr.	2000	2000	–	.005	=	.005	9	=	9	30	=	30	5%	5%
Y_3 Teaching units/yr.	50	50	–	.03	=	.03	–		–	200	=	200		

(outputs are identical for H1 and H15)

Total Inputs Required	H1	H15	Difference
FTE	23.5 (1)	26.5 (2)	3
Bed days	41,050 (3)	47,370 (4)	6,320
Supplies	$130,000 (5)	$160,000 (6)	$30,000
Total Cost	$775,500 (7)	$898,700 (8)	$123,200

*Vacancy rate reflects the % of vacant beds that exist. Beds available to exceed the number used by 1/.95 in efficient hospitals (5% are vacant). If 7 beds are used, 7/.95 must be available on average during the year.

(1) H1-FTE's = (3000 Reg. Pat.)(.004) + (2000 Sev. Pat.)(.005) + (50 Teach. Units)(.03) = 23.5

(2) H15-FTE's = (3000 Reg. Pat.)(.005) + (2000 Sev. Pat.)(.005) + (50 Teach. Units)(.03) = 26.5

(3) H1-Bed days = [(3000 Reg. Pat.)(7) + (2000 Sev. Pat.)(9)] ÷ (.95 Vacancy factor) = 39,000 ÷ .95 = 41,050

(4) H15-Bed days = [(3000 Reg. Pat.)(9) + (2000 Sev. Pat.)(9)] ÷ (.95 Vacancy factor) = 45,000 ÷ .95 = 47,368

(5) H1-Supply $'s = (3000 Reg. Pat.)(20) + (2000 Sev. Pat.)(30) + (50 Teach. Units)(200) = $130,000

(6) H15-Supply $'s = (3000 Reg. Pat.)(30) + (2000 Sev. Pat.)(30) + (50 Teach. Units)(200) = $160,000

(7) H1-Total Cost = (23.5 FTE)($10,000/FTE) + (41,050 Bed days x $10/bed day) + $130,000 Supplies = $775,500

(8) H15-Total Cost = (26.5 FTE)($10,000/FTE) + (47,368 Bed days x $10/bed day) + $160,000 Supplies = $898,700

Exhibit 1 Appendix -

Structural Model (Efficient Hospital Operations): Efficient Input-Output and Cost Relationships Assumed in the Hospital Production Model to Create the Simulated Data Base in Exhibit 2

	Amount of each Input required to Efficiently produce one unit of output.			Efficient Costs of Outputs
Outputs:	Full Time Equivalents of Labor (FTE's)	Bed days available	Supply $'s	
Regular Patient Care	.004 FTE/per patient	7 beddays/per patient	$20 supplies/per patient	$130/regular (1) patient
Severe Patient Care	.005 FTE/per patient	9 beddays/per patient	$30 supplies/per patient	$170/severe (2) patient
Training outputs	.03 FTE/per training unit	-	$500 supplies/per training unit	$500/training (3) unit

Input $ Costs for all Hospitals

FTE's	Beddays	Supply's
$10,000/FTE	$10/Bed day	$1/Supply unit

Other Assumptions:

a) Vacancy Rate - Efficient hospitals will have 5% of total beds vacant during the year (available for emergencies).

b) There are no regional cost differences for bed days, supplies, and FTE's and that the mix of FTE's and supplies are similar between hospitals.

c) Cost of unused bed days = $10/bed day.

(1) Cost/regular patient = (.004 FTE/patient)($10,000/FTE + (7 bed days/patient) ($10/bed day) + ($20 supplies/patient = $130/patient

(2) Cost/severe patient = (.005 FTE/patient) ($10,000/FTE) + (9 bed days/patient) ($10/bed day) + ($30 supplies/patient) = $170/patient

(3) Cost/training unit = (.03 FTE/training unit) ($10,000/FTE) + ($200 supplies/training unit) = $500/patient

Table 1

Efficient DMU's	DEA Efficiency Rating (E)	Efficiency Reference Set
H1	1.0	
H2	1.0	
H3	1.0	
H4	1.0	
H5	1.0	
H6	1.0	
H7	1.0	

Inefficient DMU's	DEA Efficiency Rating (E)	Efficiency Reference Set
H8	0.99	H4
H9	0.98	H2
H10	1.0	
H11	0.85	H1, H4, H6
H12	0.99	H3, H4
H13	1.0	
H14	0.99	H1, H4, H6
H15	0.87	H4, H6, H7

Application of DEA to Artificial Data Base.

DEA was applied to the 15 hospitals (DMU's) in the artificial data base developed in Ex. 2 using the three outputs (severe patients, regular patients and teaching outputs) and three inputs (fully time equivalents, bed days available, and supply dollars). The DEA efficiency rating is reported in Table 1.

DEA has accurately identified six of the eight inefficient DMU's ($E < 1$) as reported in Table 1. Two ineffecienct DMU's, H10 and H13 are rated as efficient ($E = 1$) along with the seven efficient DMU's. The six DMU's that are rated as inefficient have distinct inefficiencies present which calculated by DEA by comparison with certain efficient units that comprise an efficiency reference for the inefficient DMU (see table 1). For example, H8 was found to be inefficient by direct comparison with H4; and H15 is being compared directly with H4, H6, and H7. DEA isolates the efficiency reference set enroute to seeking the highest efficiency rating possible among the observational data subject to the constraint that no DMU can have an efficiency rating greater than 1.0. Hence, the efficiency rating and efficiency reference set are objectively determined via DEA. We defer further interpretation of the DEA information about inefficient units till section 5 and now consider the merits of DEA as an identification tool.

Two of the inefficient DMU's were not so identified. This reflects a characteristic that may arise in any DEA study. A DMU will be located as inefficient only if it is found to be relatively inefficient compared to other DMU's in the data set. Thus, DEA was not able to locate other DMU's against which H10 and H13 are found to be inefficient. Another way of describing this result is that efficient units are not necessarily efficient in an absolute sense. Indeed, some data sets may have no absolutely efficient units present. This may be viewed as a weakness of DEA.

The primary strength of DEA is that those DMU's which are rated as inefficient are relatively inefficient compared to other DMU's in the data set and the inefficiencies present can be identified by comparing each inefficient DMU with its efficiency references set as we do in section 5. Hence, DEA is reliable with respect to the inefficient units it locates but may not locate all the inefficient units present.

Ratio analysis applied to the artificial data.

We now consider how a manager might determine which DMU's are more and less efficient using ratio analysis, a widely used form of analysis to evaluate financial and operating performance. In this example, all the inputs are jointly used by these DMU's to produce three outputs. A number of different ratios might be developed to evaluate different sets of relationships such as FTE's/patient, FTE's/severe patient, FTE's/regular patient, FTE's/teaching output, bed days/patient, bed days/severe patient, etc. Such a set of ratios does not explicitly recognize the joint use of these inputs to produce these various outputs. In addition, for the set of ratios calculated, a DMU may be among the highest (least efficient) for certain ratios and lowest (most efficient) for other ratios. This leads to some ambiguity as to whether that DMU is efficient or inefficient and calls for some method of weighting or ordering the importance of the ratios to gain some overall assessment of efficiency as was generated using DEA in Table 1.

Rather than directly address this issue, we will focus on a type of unit costing ratio analysis that is typically applied to hospitals and other organizations to determine how well it performs in evaluating these DMU's. By design, all 15 hospitals (DMU's) paid the same price per unit for each type of input. Hence we can combine the inputs into dollar units without the confounding effect of differing input costs. Rather than deal with all these

Table 2

Single Output Measures

Hospital Efficient Units	Average Cost per Patient (A)		Case Mix Adjusted Average Cost per Patient (B)		Case Mix Adjusted Average Cost per Patient Segregated into High and Low Levels of Teaching Outputs Low* (C)		High* (D)	
H1	$155.10	(2)	$138.48	(4)	$138.48	(4)		
H2	163.32	(5)	138.40	(3)	138.40	(3)		
H3	168.32	(7)	142.65	(8)			$142.65	(3)
H4	160.10	(4)	142.94	(9)	142.94	(2)		
H5	158.38	(3)	137.73	(2)			137.73	(2)
H6	170.15	(9)	140.12	(5)	140.12	(1)		
H7	142.60	(1)	135.81	(1)			135.81	(1)
Ineffficient Units								
H8	176.95	(11)	157.99	(12)**			157.99	(6)
H9	168.32	(7)	142.64	(7)	142.64	(6)		
H10	169.69	(8)	161.61	(14)**			161.61	(7)
H11	170.33	(10)	153.10	(10)	153.10	(7)		
H12	178.33	(12)	155.07	(11)			155.07	(5)
H13	165.68	(6)	142.00	(6)	142.00	(5)		
H14	178.33	(12)	155.07	(11)			155.07	(5)
H15	179.74	(13)	160.48	(13)**	160.48	(8)		
Mean	167.02		146.94		144.77		149.42	
Standard Deviation			8.82		7.36		9.66	

* Low teaching outputs were 50 units and high teaching outputs were 100 units as per Exhibit 3, Col. 6.

**Hospitals more than one standard deviation over average cost.

outputs, the teaching output might be viewed as a by-product or secondary output and the patients might be viewed as a single output rather than segregate this into different categories of severity. This simplifying procedure is not defensible from a cost accounting standpoint. Nevertheless, in the absence of any other way of combining and weighting the outputs, similar approaches have been used for hospitals as well as other types of DMU's (see for example [20]).

Table 2 column (A) reflects the average cost per patient for each DMU. This results in a ranking of hospitals reflected by the rank order in parenthesis directly to the right of the average cost figure in Table 2. The lowest cost (most efficient) DMU is ranked 1 and highest cost (least efficient) DMU is ranked 13. This ranking erroneously classifies H13 (ranked 6) as more efficient than H3 (rank 7) and H6 (rank 9) and it classifies H9 as more efficient than H6. In addition, there is no objective means for determining the cutoff cost level to segregate efficient and inefficient units.

If the efficient relative costs of certain outputs are known, the outputs can be weighted to reflect a cost per weighted output units. In this case we know the efficient cost of a regular patient ($130) and a severe patient ($170) and the patient units can therefore be weighted to value each severe patient as the equivalent of 170/130 ~ 1.3 regular patients. For example, H1 would have adjusted patient output units of 3000 regular patients + 2000 x 1.3 severe patients for an adjusted total of 5600 patients. This results in an adjusted average cost per patient of $130.40 compared with the unadjusted cost of $155.10 for H1.

The adjusted cost per patient is reflected in column (B) of Table 2 with the new ranking in parenthesis immediately to the right of the average cost per day. Again, we have a misranking with inefficient DMU's H9 and H13 being ranked as more efficient than H3 and H4.

If we further segregate the 15 DMU's by the third output (teaching) and separate them based on those with high (100 units) versus low (50 units) teaching outputs, the ranking based on unit costs is reflected in columns C and D in Table 2. At this point, we have achieved an accurate ranking, but again we have no objective method of separating efficient from inefficient DMU's.

The problem of locating a point beyond which DMU's are considered inefficient is typically addressed by establishing a subjective cutoff point, but there is no assurance that the inefficient units will be accurately located through this process. For example, if the cutoff was set at one standard deviation above the mean adjusted cost per patient, only 3 DMU's (H8, H10 and H15) would be identified as inefficient as indicated in column (B) of Table 2.

A unit cost analysis of the type completed above could not have been completed if the efficient cost or efficient relative cost of various outputs were not known. Such information is frequently absent in non-profit settings such as health and education. In addition, if the cost of inputs varied among the DMU's, this technical efficiency analysis would be confounded by the DMU's purchasing behavior which might lead to other inaccuracies in use of such ratios. Thus, ratio analysis applied in this unit costing manner required additional information about the production and cost relationships beyond that required for DEA and still provided a less objective method of identyfying the inefficient DMU's than that achieved using DEA, since the cutoff of efficient vs inefficient is not provided from the ratio analysis.

Regression analysis applied to the artificial data base.

In industries where the efficient input-output relationship i.e., the technology is not known with any precision, regression analysis has been used

to gain insights into the production relationships that exist among the observed data. Nevertheless, it is not clear how traditional regression analysis can be used to evaluate the efficiency of individual DMU's. The primary problem is that the regression estimates of production and cost relationships are based on least squares estimates which provide a mean or central tendency set of relationships and therefore reflects a mixture of efficient and inefficient behavior in the data set. Thus, regression relationships will only reflect efficient relationships if all units in the study are themselves efficient. In a non-profit setting, such as in education, health, and government, where it is difficult to justify the assumption of efficient behavior due to lack of competition, in an economic sense, the resulting least squares regression may not reflect efficient relationship.[7]

We now consider the extent to which regression analysis can be used to identify the inefficient units in the artificial data set. We again take advantage of the knowledge that all DMU's pay the same prices for all inputs and attempt to develop an estimate of the total cost as a function of the outputs produced by each DMU.

We consider a simple additive (linear) regression model and multiplicative (nonlinear) model to determine what insights are available about the production function and about the efficiency of the 15 DMU's.

Additive Regression Model

Total cost was estimated as a function of the quantity of three outputs produced by each DMU. The results were as follows:

$$C = -95.300 + 152\ y_1 + 182.4\ y_2 + 1302\ y_3$$
$$\quad (8) \qquad (22.2) \qquad (767) \tag{9}$$

where C = Total cost per year

y_1 = # of regular patients treated per year

y_2 = # of severe patients treated per year

y_3 = # Training units provided in one year.

The high R^2 value of .97 suggests a good fit with the observational data, so by standard reasoning, a high degree of cost variation is explained by these independent variables. The expression yields a fixed negative cost estimate of $95,300, although no such costs appear in the underlying model.

The estimated incremental cost of these outputs versus the actual or efficient cost of the outputs is summarized below:

Output	Estimated incremental Cost	Actual (efficient) incremental Cost
y_1	$ 152.	$130
y_2	$ 182.40	$170
y_3	$1302.	$500

The incremental costs differ from the true costs and, indeed, the estimated cost for y_3 is very wide of the mark.

The expression (9) represents a widely used form of cost via regression approach subject only to evaluation of the economic reasonableness of the relationships and standard tests of significance and independence of right hand side variables.[8] Although the estimates may differ from the actual marginal costs, such a regression may be useful for other purposes e.g., for pure prediction due to the high R^2. The use of the coefficients in (9) to

estimate marginal costs is dubious and so would be its use in evaluating the efficiency of individual hospitals. One approach which might be employed is to designate inefficient DMU's as those for which actual total cost is some arbitrarily determined distance above the estimated total cost. In this example, inefficient DMU's H8 and H13 have actual cost which are near or below the estimated costs and would not be considered inefficient based on this rule to locate the inefficient DMU's. All the efficient DMU's have actual costs below the regession estimate and the other six inefficient DMU's actual costs are above the regression estimate. Hence, in this case, the use of this regression estimate with the identification rule described above as a basis for locating inefficient units appears to be more accurate than ratio analysis, and as accurate as DEA, in that it located six inefficient DMU's. Again, the rule to segregate efficient and inefficient that yielded this result units was arbitrarily established.

Nonlinear regression model

A nonlinear regression model might be used instead of the additive model because it allows for the possibility of returns to scale. The use of a multiplicative regression model was applied to the 15 DMU's with the following results:

$$\ln C = 3.98 + \underset{(.04)}{.62} \ln y_1 + \underset{(.07)}{.57} \ln y_2 + \underset{(.05)}{.10} \ln y_3$$

or (10)

$$C = 53.79\, Y_1^{.62}\, Y_2^{.57}\, Y_3^{.10}$$

In (21), the sum of the coefficients (.62 + .57 + .20) exceeds 1 which suggest the presence of decreasing returns to scale i.e., doubling each output will more than double the total cost. In addition, there are partial scale economies for increases in each output individually holding other outputs constant. We know that the underlying cost function is linear in these outputs so that neither of these effects are present. Hence, in spite of the high R^2 value of .96, this nonlinear cost function also does not mirror the true relationship and conclusions about efficient cost behavior drawn from application of regression techniques to a set of efficient and inefficient DMU's can be misleading.

If we were to arbitrarily consider DMU's for which actual total cost exceeds the estimated total cost in (10) as the potentially inefficient units, then efficient DMU's H2, H6, and H7 would be erroneously considered inefficient and inefficient DMU's H11, H12, H13, and H14 would be identified as efficient. Hence, in this example the result of the nonlinear regression would be inferior to the additive regression results, the ratio analysis and DEA results. Although the nonlinear regression results are less meaningful than the linear regression, it is not clear that we could objectively reject the nonlinear results in favor of the linear regression results, since both have high R^2 values and we are attempting to locate inefficient units without knowing that the underlying production and cost relationships are linear *a priori*. Hence, our ability to identify inefficient DMU's will be influenced by the regression model we select. Other more sophisticated regression techniques which also reflect mean relationships rather than the extermal efficient relationships also prove inadequate for purposes of effecting separation between efficient and inefficient units.[9]

Table 3

Comparison of DEA, ratio analysis, and linear regression approaches ability to locate Inefficient DMU's

E = DMU rated as efficient
I = DMU rated as inefficient

Efficient DMU's	(A) DEA Results (1)	(B) Ratio Analysis (2)	(C) Regression Linear Model (3)	(D) Analysis Non linear Model (3)
H1	E	E	E	E
H2	E	E	E	I
H3	E	E	E	E
H4	E	E	E	E
H5	E	E	E	E
H6	E	E	E	I
H7	E	E	E	I
Inefficient DMU's				
H8	I	I	E	I
H9	I	E	I	I
H10	E	I	I	I
H11	I	E	I	E
H12	I	E	I	E
H13	E	E	E	E
H14	I	E	I	E
H15	I	I	I	I

(1) From table 1

(2) From table 2 column B - DMU's with cost/patient greater than one standard deviation above the mean.

(3) Based on rule that DMU's with actual total cost greater than estimated total cost (based on the regression model) are inefficient

Table 3 summarizes the results of the three methodologies in locating inefficient units. We found that DEA located six of the eight inefficient DMU's as did the linear regression model when the rule that DMU's with actual costs above the mean (ratio analysis) are deemed inefficient (columns A and C in table 2). Ratio analysis located only 3 of the inefficient DMU's (Col. B of Table 3). Finally, the nonlinear regression approach proved far less reliable (see table 2, col. D) in that 3 efficient DMU's were identified as inefficient.

Based on these results, DEA analysis may be argued to be the more objective and reliable in locating inefficient DMU's for the following reasons.

1. Ratio analysis and regression analysis required an arbitrary rule to determine which DMU's would be designated as inefficient. With ratio analysis, the mean might well have been lower or higher depending on whether there were more or less efficient units in the data set. Similarly, regression analysis might also have a lower or higher cost curve depending on the relative number of inefficient units.

2. Ratio analysis required added relative price data and other iterations to address the multiple output and input situation while DEA could address this directly using only physical output and input data. In addition, the ratios could easily be confounded if DMU's paid different prices for similar outputs. For example, a DMU that had very low prices might have a lower average cost which obfuscates the presence of technical inefficiency that could lead to further reductions in inputs and costs. Regression analysis also assumed DMU's had the same costs/input, as different cost structures would have shifted the cost function and produced different but not necessarily more accurate results.

3. Regression analysis results depended on the selection of an appropriate model or set of cost relationships. In both the linear and nonlinear case, cost relationships were misleading and in the use of the nonlinear model, the identification of inefficient DMU's was the least accurate of all these approaches.

4. The added information about the nature of inefficiencies with ratio analysis was negligible and with regression analysis it was misleading.

In the following section, we consider the extent to which DEA results provide insights into the nature of the inefficiencies located as well as consider other limitations and strengths of DEA that need to be considered in assessing DEA versus ratio analysis and regression analysis for identifying

inefficient DMU's.

4. Interpretation of DEA results

The application of DEA to this artificial data set highlights two key areas that need to be considered by a manager using DEA: 1) data specification and 2) interpretation of the efficiency ratings.

Data Specification

The DEA results reliably located six of the eight inefficient DMU's in this application partly because the data were specified and measured in a manner consistent with the way the structural model was developed. That is, data in the form of physical inputs and outputs were incorporated in the DEA efficiency evaluation. In the absense of knowledge of the true structural model, it may be possible to specify input and output data in more than one reasonable and relevant way, but such specifications may not be consistent with the true production function and may result in somewhat less reliable DEA results.

For example, if the output measure used in the simulation was patient days instead of <u>number</u> of patients, the DEA results would identify DMU's that were inefficient with respect to patient <u>days</u> of output. For example, using patient days as an output measure, the DEA results would also identify inefficient DMU H8 as efficient. This result arises because H8 was not inefficient with respect to how many patient days of output provided. H8 was only inefficient with respect to how many days per patient it utilized. Hence, the DEA results using a patient day output measure would yield an accurate conclusion with respect to the output measure used, but the result would not necessarily be accurate with respect to the true production function. This suggests that alternative output measures need to be considered where there is ambiguity as to what input and output measures should be used for evaluating the DMU's under study.

Another related issue is the way data is specified. In the simulated application, data in the form of physical input and output units were available. In many applications, data may include a mixture of physical measures and indices. For example, a breakdown of how many patients were severe and how many were regular (not severe) may not be available and the data may only be available in the form of a total number of patients treated accompanied by an index of the severity of the patients. When DEA was applied to this data set with outputs specified as total patients, percent severe (an index), and teaching units (instead of the regular patient, severe patient, and teaching units) the results proved less reliable. Efficient hospitals H5, H6, and H7 were identified as inefficient along with all the other inefficient DMU's. This again suggests that a sensitivity analysis to alternative data specifications is needed before relying on the DEA results and particularly where data is not specified in the form of physical units of inputs and outputs.[10]

Interpretation of the efficiency rating.

The efficiency rating E used in this application of DEA defined in (8) is useful primarily to indicate whether the DMU is inefficient i.e., $E < 1$. The actual value of E will depend on the value assigned to ε in (7). For example, the efficiency rating for inefficient hospital H11 was .8527 with $\varepsilon = .001$. This efficiency rating increases to .9853 with $\varepsilon = .0001$ and to .99853 with $\varepsilon = .00001$.[11] While a natural tendency for any researcher in the social sciences would be to round this last efficiency rating to 1.0, this rating of .99853 is distinct from 1.0 in this type of analysis and indicates the presence of inefficiencies in H11. This is illustrated in table 4 where the output of the linear program results of the DEA application is reported for H11 on the lower half of the table and an interpretation of these results is presented above that.

Table 4 - Hospital 11 DEA Results

efficiency rating = .853   .9853   .99853
Hospital 11 ε = .001 ε = .0001 ε = .00001

Input Output Vector	(A) Objective Hospital H11	(B) Efficiency Reference Set H4	(C) Efficiency Reference Set H7	(D) Efficiency Reference Set (1.43)(B) + (.071)(C)	(E) H11 Compared with composite (D) - (A)
Inputs					
x_1 FTE's	44.5	25	51.5	39.4	5.1
x_2 Bed Days	65,260	41,050	92,630	65,260	0
x_3 Supply $'s	$265,000	$140,000	$270,000	$219,290	$45,710
Outputs					
y_1 Teaching	50	100	50	146.4	96.4
y_2 Severe Patients	3,000	2,000	2,000	30.00	-
y_3 Regular Patients	5,000	3,000	10,000	5,000	-

Shadow prices Related To The LP Constraints*

H4 = 1.43

H7 = 0.071

$v_1 = -5.11$

$v_2 = 0$

$v_3 = -45.71$

$u_1 = -96.4$

$u_2 = 0$

$u_3 = 0$

*same value for ε =.001, .0001, and .00001.

The (non zero) shadow prices related to the DMU constraints in the linear program formulation in (6) indicate the efficient DMU's against which the inefficient DMU is being most directly evaluated. We refer to these efficient DMU's that form the basis for the efficiency rating of inefficient DMU's as the efficiency reference set and these are listed in table 1 for each inefficient DMU. In this example, H11 is being compared directly with an efficiency reference set of DMU's H4 and H7. The shadow prices reflect the weights assigned by the DEA in determining the relatively efficient point against which H11 was compared. This is illustrated by the columns B, C, and D in table 4 where the input/output vector for DMU's H4 and H7 are each weighted by their respective shadow prices and summed to yield a composite of these two efficient DMU's. This composite of 2 efficient DMU's is specifically more efficient than H11. Column E in table 4 indicates that H11 produced 96.4 fewer units of y_1 (and the same amount of y_2 and y_3) than the composite (col. D) and used 5.1 more units of x_1 and 45,710 more units of x . Thus, H11 is less efficient than a combination of two other observed units.

The amount of inefficiency located in table 4 represents a type of information about the magnitude and possible location of the inefficiency which distinctly augments the efficiency rating information. The presentation in table 4 provides one direct and managerially understandable way to comprehend the magnitude of inefficiences indicated by the efficiency rating E. Table 4 is an example of how the inefficience located via DEA are distincly and objectively located based on direct comparison with other DMU's in the data set.

The information provided in table 4 must also be qualified with respect to the degree to which it can be literally interpreted. DEA results in table 4

directly indicate that a combination of two DMU's operating results would produce a composite DMU that is more efficient than H11. This indicates one way for H11 to become more efficient. This is not, however, the only direction that H11 can choose or should choose to improve efficiency. For example, H11 was inefficient with respect to its use of FTE's and supplies used to provide severe and regular patients care (see exhibit 2). The adjustment required to make H11 efficient based on its actual outputs and the structural model differs from the adjustment suggested in table 4 as is indicated in table 5:

Table 5

	A Actual Inputs and outputs of H11	B Efficient Inputs for H11 based on structural model	C Adjustment Required to become efficient (B - A)	D Adjustment indicated from table 4 based on DEA results
x_1	44.5	36.5	-8	-5.1
x_2	65,260	65,260	0	0
x_3	265,000	200,000	-$65,000	-45,710
y_1	50			+96.4
y_2	3000			0
y_3	5000			0

Table 5 compares the true adjustment required by the structural model for H11 to become efficient (column C) with the DEA results (Column D). In other words, Column C in table 5 reflects the adjustments required for H11 to exactly fit the structural model. The two sets of adjustments in Columns C

and D are both mathematically accurate ways for H11 to become efficient. Note however, that only the solution in Column D is available directly from DEA and that the "true" solution in Column C is not available. In addition, it may be impractical or impossible to make the adjustments suggested by Column D in table 5 i.e., can H11 actually increase teaching outputs to nearly three times its recent level? This suggests that the manager might use the DEA results to indicate of the presence of inefficiency and possible areas where the inefficiencies lie. The DEA results suggest alternative paths to improve efficiency e.g., in the above case, H11 could either emulate H4 or H7 or it could aim for the composite input-output level suggested in table 4. The identification of preferred and attainable paths to improve efficiency would naturally be based on managerial judgment. Should this lead to proposed paths that differ from the one derived from DEA, it is also possible to reapply DEA for a sensitivity analysis to determine if other paths proposed by management would improve the efficiency compared to the other DMU's in the data set.

5. Conclusion

The simulated application of various efficiency measurement techniques to an artificial data set to evaluate DMU's with multiple outputs and inputs appears to support the following conclusions:

DEA versus single regression analysis

DEA more objectively located inefficient DMU's than simple regression techniques without the need to collapse the inputs by relative prices the need to specify a functional form. This result is not surprising since DEA is a methodology that was developed as an efficiency evaluation technique and simple regression analysis reflects mean or central tendency relationships

which is more appropriate for pure prediction (assuming the level of inefficiency among the DMU's remains at the level in the observational data) than for ascertaining efficient input-output relationships.

DEA versus ratio analysis

DEA more objectively located inefficient DMU's than ratio analysis. Ratio analysis cannot directly evaluate multiple input-multiple output DMU's , as the ratios are of a single input per single output form. While ratio analysis is widely used to gain insights into certain operating relationships, it nevertheless has severe limitations in dealing with the multiple output-multiple input case particularly in the absense of an objective weighting system that can be used to collapse these outputs and inputs into a single output to single input ratio. After using added information to develop a unit costing ratio, an arbitrary rule had to be adopted to identify inefficient DMU's with ratio analysis.

Use of combined techniques to evaluate efficiency

Efficiency evaluations may be most effectively accomplished by a combination of methodologies. For example, DEA might be used to select out certain of relatively inefficient DMU's after which regression analysis could than be applied to the set of more efficient DMU's to more accurately estimate the true marginal costs of outputs. At the same time, DEA was capable of locating relatively inefficient DMU's and their efficiency reference sets i.e., relatively efficient DMU's against which the inefficient DMU's were being most directly compared. This provided certain insights about alternative paths for improving the efficiency of relatively inefficient DMU's.

The DEA results pointed to various paths some of which may not be feasible ways for an inefficient DMU to improve operations. Once DEA has reduced the DMU's of interest to the set of inefficient DMU's and their efficiency reference set, analytical techniques like ratio analysis may be helpful at locating the sources of the inefficiencies and the paths that may be most attainable or appropriate for that DMU.

Generalizability

The above conclusions arise from simulated application of alternative methodologies to a simple data set. The problems encountered with ratio analysis and simple regression analysis are clearly ones that argue against their usage to evaluate efficiency of multiple output-multiple input DMU's. We have indicated that when DEA locates an inefficient DMU this determination is objectively verifiably compared to other DMU's in the data set (see table 5). This attribute provides grounds for optimism about DEA's usefulness. Nevertheless, the types of problems which may arise from misspecification of the data e.g., use of indices or use of physical input and output measures that are inconsistent with the undelying technology, suggests that the sensitivity to these specification must be carefully considered before one can rely on the results for managerial purposes.

Directions for further DEA research

DEA has been shown to be mathematically sound and consistent with economic theory by Charnes, Cooper, and Rhodes [9] [11]. In addition, it has been found to be reliable at locating inefficient DMU's in this simple applications to an artificial data set. Further examination of the DEA's limitations

located in this study is needed and might include research into the following questions:

a. How can data specification for DEA applications and sensitivity analysis using DEA be most effectively accomplished to achieve results which are most consistent with the underlying technology?

b. Are there statistical techniques which might enable one to assess DEA's reliability when data cannot be specified in physical input and output units and specifically how can indices be accomodated in the DEA analysis?

c. Can DEA results be used to gain insight into the efficient rates of input and output substitution and the optimal path to improve efficiency?

d. In the data base used in this study, we assumed no substitutability among inputs and outputs and constant return to scale. How well would DEA perform if these assumptions were relaxed?

e. Are there relationships between the number of inputs and outputs and the number of observations that would allow one to estimate the degree to which the inefficient DMU's will escape identification using DEA.

Overall, the results of this study suggest the DEA is a promising tool to evaluate relative technical efficiency of DMU's with multiple outpus and inputs where the efficient production function is not specifiable with any precision. DEA is also being applied to real data sets with apparent acceptance and expanded DEA models to more directly deal with economies of scale are under development.[12] At this point, we feel that further validation of DEA in field and simulated settings is needed to better understand the strengths and limitations and to support the expanded use of DEA by management in other non-profit, public sector, and for profit settings.

FOOTNOTES

1 We are primarily concerned with technical efficiency as distinct from allocative or price efficiency which relates to the question of whether a DMU is using an optimal mix of inputs and purchasing them at the lowest price. A DMU is technically inefficient if it can produce more outputs with the amount of inputs it used or can produce its level of outputs with fewer inputs than were used. Hence, a technically inefficient unit can reduce inputs and attain the same level of outputs regardless of any other types of efficiencies or inefficiencies that may be present.

2 For example, regression analysis has been applied to health care organizations in numerous studies, e.g., Feldstein [14], and Zaretsky [23] and ratio analysis has been used by health regulatory organizations to locate inefficient hospitals as in [20].

3 See Sherman [22] for a discussion of hospital efficiency and the need for improved hospital evaluation techniques.

4 The original form of the fractional linear program developed by CCR [9] is as follows:

Objective:

$$\max h_o = \frac{\sum_{r=1}^{s} \mu_r y_{ro}}{\sum_{i=1}^{m} w_i x_{io}}$$

<u>Constraints:</u>

Less than Unity Constraints : $1 < \dfrac{\sum_{r=1}^{s} \mu_r y_{rj}}{\sum_{i=1}^{m} w_i x_{ij}}$; $j = 1,\dots,n$

Positivity Constraints: $0 < \mu_r$; $r = 1,\dots,s$
$0 < w_i$; $i = 1,\dots,m$

<u>Data:</u>

Outputs: y_{rj} = observed amount of r^{th} output for j^{th} hospital

Inputs: x_{ij} = observed amount of i^{th} input for j^{th} hospital

CCR in [9] describe the way the form we use in (6) may be derived from the fractional form.

This methodology builds on concepts developed by Farrell [13] and Carlson [7].

5 We performed a series of checks to determine if smaller values of ε alter the solution. As will be discussed at a later point in the paper, smaller ε values do not change the interpretation and conclusions of the DEA analysis. Similar tests must be performed for any DEA application to assure that the ε value is sufficiently small.

6 See CCR [9] for further discussion of these slack values and their interpretation. It is also possible to obtain the same understanding of which units are inefficient by using $\varepsilon = o$, which circumvents the need to assign a small nonzero value to ε. In this case, a DMU with a value of $h_o^* = 1$ is efficient if and only if there is zero slack and a DMU is inefficient if $h_o^* = 1$ and there is positive slack or if $h_o^* < 1$. We prefer to assign a small value to ε as it enables one to determine which DMU's are inefficient by reference only to the E (efficiency rating) value and because it has been found to be more easily understood and used by managers who need only use the simple rule that $E = 1$ is efficient and $E < 1$ is inefficient. The expression (18) is clarified by example in section 5 and in footnote 11.

7 Similar problems exist in the for profit corporate sector, though existence of free market competition would suggest that this is a less serious issue since firms may be assumed to be moving toward more efficient behavior. Nevertheless, it would be naïve to assume that in any real data set, all DMU's are efficient so there will tend to be problems in evaluating efficiency based on simple regression results.

8 The independent variables were found to have very low correlation coefficients as follows:

$$y_1, y_2 = -.37$$

$$y_1, y_3 = -.03$$

$$y_2, y_3 = -.08$$

Hence, the problem in the coefficient values is not due to multicollinearity.

9 See for example Sherman [22] where the flexible functional from translog function is found to lead to misleading results. Other types of regression techniques which attempt to estimate extremal relationships that more closely approximate efficient production or cost functions have been proposed.Forsund, Lovell, and Schmidt [15] review a variety of these extremal approaches and suggest that as currently constructed, they have severe limitation due to the assumptions required in their use. See also [1], [2], and [18].

10 See Sherman [22] for an extended discussion of why these DEA results shift when indices are used to replace data measured in physical units.

11 Specifically, the value of E for H11 with ε = .001 is calculated as follows from (8):

$$E = h_o^* - \varepsilon \left[\sum_{r=1}^{3} s_r^{-*} + \sum_{i=1}^{3} s_i^{+*} \right]$$

$$= 1.0 - .001\ [5.11 + 45.71 + 96.4] \sim .853$$

The slack values, s_r^{-*} and s_i^{+*}, for H11 are reported in table 4.

12 See for example Banker, Charnes, Cooper, and Schinar [3].

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